

# On Volume Stabilization by Quantum Corrections

Marcus Berg<sup>†</sup>, Michael Haack<sup>†</sup> and Boris Körs<sup>\*</sup>

<sup>†</sup>*Kavli Institute for Theoretical Physics, University of California  
Santa Barbara, California 93106-4030, USA*

<sup>\*</sup>*Center for Theoretical Physics, Laboratory for Nuclear Science  
and Department of Physics, Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139, USA*

<sup>\*</sup>*II. Institut für Theoretische Physik der Universität Hamburg  
Luruper Chaussee 149, D-22761 Hamburg, Germany*

<sup>\*</sup>*Zentrum für Mathematische Physik, Universität Hamburg  
Bundesstrasse 55, D-20146 Hamburg, Germany*

## Abstract

We discuss prospects for stabilizing the volume modulus of  $\mathcal{N} = 1$  supersymmetric type IIB orientifold compactifications using only perturbative corrections to the Kähler potential. Concretely, we consider the known string loop corrections and tree-level  $\alpha'$  corrections. They break the no-scale structure of the potential, which otherwise prohibits stabilizing the volume modulus. We argue that when combined, these corrections provide enough flexibility to stabilize the volume of the internal space without non-perturbative effects, although we are not able to present a completely explicit example within the limited set of currently available models. Furthermore, a certain amount of fine-tuning is needed to obtain a minimum at large volume.

# 1 Introduction

In a companion paper [1] we computed one-loop corrections to the Kähler potentials of certain supersymmetric orientifolds of type IIB string theory. Here, we interpret these results in the context of volume stabilization. The two papers can be read independently.

To make contact with phenomenology, a viable string theoretical model must deal with the problem of stabilizing the scalar moduli fields. One way to generate a potential for many of these scalars is by turning on background fluxes. In type IIB orientifolds, fluxes lead to *no-scale* potentials, which stabilize some moduli but leave the Kähler moduli unfixed, among them the overall volume of the internal space [2]. The general form of this flux-induced potential is given by the standard F-term expression of  $\mathcal{N} = 1$  supergravity<sup>1</sup>

$$V = e^K \left[ K^{M\bar{N}} D_M W D_{\bar{N}} \bar{W} - 3|W|^2 \right] , \quad (1)$$

where the superpotential only depends on the complex structure moduli  $U^\alpha$  and the axio-dilaton  $S$ ,

$$W_{\text{flux}} = W(S, U^\alpha) . \quad (2)$$

Here  $M, N$  run over all the fields (chiral multiplets) of the theory. The Kähler covariant derivative is defined as usual,  $D_M W = (\partial_M + K_M)W$ , and subscripts indicate derivatives.

For a IIB string compactification on a Calabi-Yau manifold with a single Kähler modulus  $\rho$  whose imaginary part measures the overall volume of the internal space, the classical Kähler potential for  $\rho$  has the form<sup>2</sup>

$$K(\rho, \bar{\rho}) = -3 \ln(\rho - \bar{\rho}) . \quad (3)$$

The expression is “classical” in the sense that it is leading order in string perturbation theory and in the  $\alpha'$  expansion of the effective action. In particular, the moduli space factorizes and the Kähler metric is block-diagonal with one factor for the Kähler moduli. The superpotential (2) and the Kähler potential (3) together satisfy

$$K^{\rho\bar{\rho}} D_\rho W D_{\bar{\rho}} \bar{W} = 3|W|^2 , \quad (4)$$

---

<sup>1</sup>We always refer to IIB orientifolds with D3- and D7-branes here. In models with D5- and D9-branes, D-term potentials appear as well [3].

<sup>2</sup>We leave out additive constants in writing the Kähler potential, such as  $-3 \ln[-i]$  in this case, since they do not affect the Kähler metric, but we do take care of these factors in the prefactor  $\exp(K)$  of the potential (1).

which leads to the no-scale property [4] of the classical potential of orientifold flux compactifications. This generalizes to several Kähler moduli  $\rho^\alpha$  and to any perturbatively generated superpotential, since the real parts (RR scalars) of the Kähler moduli obey a shift symmetry that forbids their appearance in the superpotential to all orders in perturbation theory [5, 2]. The resulting scalar potential fails to produce masses for the Kähler moduli, and thus leads to the problem of volume stabilization at the level of the leading perturbative approximation.

One is then led to go beyond the leading approximation, to break the no-scale structure of the potential, thereby generating a potential also for the Kähler moduli. There are several ways to do this. The first is to allow the superpotential to depend on the Kähler moduli. This approach was pursued by Kachru, Kallosh, Linde and Trivedi (KKLT) in [6]. Due to the  $\text{Re}(\rho^\alpha)$  shift symmetries mentioned above, the construction of [6] has to rely on non-perturbative strong coupling effects on the world volume of D-branes (gaugino condensation on D7-branes or D3-brane instantons) to produce a superpotential that depends on the Kähler moduli. This leads to a scalar potential with enough structure to stabilize also Kähler moduli. Of course, such non-perturbative effects are hard to control. Furthermore, some fine-tuning of the various input parameters is required to produce satisfactory minima at large volume.

The second way to break no-scale structure does not rely on non-perturbative strong coupling phenomena. The Kähler potential (3) naturally receives corrections in perturbation theory, both from  $\alpha'$  corrections and from string loops. These corrections break the factorization of the total moduli space (into Kähler moduli times complex structure and axio-dilaton) and lead to more complicated dependence of the Kähler potential on the Kähler moduli. As a consequence, the no-scale structure of the potential is lifted, and it is no longer impossible to stabilize the volume modulus simply by perturbative corrections to the Kähler potential, i.e. without any dependence on the volume in the superpotential.<sup>3</sup>

For  $\alpha'$  corrections, the breaking of no-scale structure was demonstrated in [11]. Unfortunately, the corrections to the no-scale potential from purely sphere-level  $\alpha'$  corrections are not sufficiently well understood to claim stabilization of the volume modulus. Here, we will add type IIB orientifold one-loop corrections that we deter-

---

<sup>3</sup>An alternative approach was considered in [7, 8, 9], combining the effects of a non-perturbative superpotential with non-trivial dependence on  $\rho$  and those of the  $\alpha'$  corrections. The result reported there was that the perturbative corrections cannot be neglected and that a class of non-supersymmetric vacua with large volume can be found. Also, [10] considered volume stabilization by perturbative string corrections in nonsupersymmetric models.

mined in [1] for certain models. We will see that they constitute another source of no-scale breaking and introduce additional structure sufficient to stabilize the volume modulus, at least in principle.<sup>4</sup> As it turns out, in the very limited set of examples analyzed in [1] a significant amount of fine-tuning is required to stabilize the volume large enough so that one can neglect further corrections. Nevertheless, we consider the results promising, in that one can now look for concrete models where the volume modulus can be stabilized by perturbative corrections alone.

## 2 General remarks on the large volume expansion

In terms of string perturbation theory, the no-scale Kähler potential (3) for the volume modulus  $\rho$  arises from sphere diagrams, expanded to leading order in  $\alpha'$ . It receives a host of corrections beginning already at disk level, i.e. tree-level of the open string. At disk level, it is known that the argument of the logarithm in (3) is shifted by a function of the open string scalars that we collectively denote  $A$  and on the complex structure moduli  $U$ , cf. [14, 15, 16, 17], in the form

$$\begin{aligned} K(\rho, \bar{\rho}) &= -\ln[\rho - \bar{\rho}] \\ \xrightarrow{\text{disk}} K(\rho, \bar{\rho}, A, \bar{A}, U, \bar{U}) &= -\ln[\rho - \bar{\rho} + f(A, \bar{A}, U, \bar{U})] , \end{aligned} \quad (5)$$

This can be considered the classical tree-level Kähler potential at leading order in  $\alpha'$ .

Beyond leading order in  $\alpha'$ , there is a correction known at order  $(\rho - \bar{\rho})^{-3/2}$  [11] whose coefficient is proportional to the Euler number of the internal Calabi-Yau manifold. Beyond leading order in the string coupling, the corrections in [1] are one-loop corrections from Klein bottle, annulus, and Möbius strip diagrams, that are typically suppressed by  $(\rho - \bar{\rho})^{-1}$  and  $(\rho - \bar{\rho})^{-2}$ , with coefficients that depend on the complex structure and axio-dilaton as well as on open string moduli. All of these corrections to (3) ruin the factorization of the moduli space (the Kähler moduli space is no longer separate), and break the no-scale structure.

Putting these pieces together, the expansion of the Kähler potential involves terms of the form<sup>5</sup>

$$\begin{aligned} K &= -3 \ln [\rho - \bar{\rho} + f_1(A, \bar{A}, U, \bar{U})] + \frac{1}{\rho - \bar{\rho}} \left[ \frac{f_2(A, \bar{A}, U, \bar{U})}{S - \bar{S}} + \dots \right] \\ &\quad + \frac{1}{(\rho - \bar{\rho})^{3/2}} [\alpha(S - \bar{S})^{3/2} + \dots] + \frac{1}{(\rho - \bar{\rho})^2} [f_3(A, \bar{A}, U, \bar{U}) + \dots] + \dots , \end{aligned} \quad (6)$$

---

<sup>4</sup>This was also discussed in [12]; see [13] as well.

<sup>5</sup>See also [18, 19] for related discussions.

where we omitted  $\rho$ -independent terms and those that are more suppressed in the large volume limit. The dots in the brackets involve terms with more inverse powers of the dilaton field  $S - \bar{S}$ . These arise from corrections above one-loop, that are suppressed at small string coupling. Without fluxes, there are no further  $\alpha'$  corrections at sphere level. But with fluxes, there could be terms leading in the dilaton expansion compared to the terms shown in (6). For example, the term suppressed by  $(\rho - \bar{\rho})^{-2}$  might receive corrections already at sphere level, from a term proportional to  $G_3^2 R^3$  (with  $G_3$  the 3-form flux). In addition, in the presence of D-branes, there may be  $\alpha'$  corrections from disk diagrams that could contribute to (6). Both of these potential additional  $\alpha'$  corrections are poorly understood and we will limit our discussion of  $\alpha'$  corrections to the  $(\rho - \bar{\rho})^{-3/2}$  term in (6).

We now consider a superpotential of the form (2) and “integrate out”  $S$  and  $U$  by setting  $D_S W = D_U W = 0$ . This fixes  $S$  and  $U$  at  $S^{(0)} + \mathcal{O}((\rho - \bar{\rho})^{-1})$  and  $U^{(0)} + \mathcal{O}((\rho - \bar{\rho})^{-1})$ , with constants  $S^{(0)}$  and  $U^{(0)}$ , and we further assume that this is a minimum of the potential with respect to  $S$  and  $U$ . The idea behind this hierarchal integrating-out is an assumption of a separation of scales, where the complex structure and the axio-dilaton receive masses much larger than the typical scale of the potential for  $\rho$  and  $A$ . Substituting the Kähler potential of (6) into (1), the scalar potential takes the form

$$V = e^K \left( K^{\bar{\rho}\rho} K_{\bar{\rho}} K_{\rho} + K^{\bar{\rho}A} K_{\bar{\rho}} K_A + K^{\bar{A}\rho} K_{\bar{A}} K_{\rho} + K^{\bar{A}A} K_{\bar{A}} K_A - 3 \right) |W|^2. \quad (7)$$

The breaking of the no-scale structure is manifest whenever the factor in parenthesis no longer vanishes. However, computing the potential using (6) one finds that there is still no term of order  $(\rho - \bar{\rho})^{-1}$  in the large-volume expansion, even though there is such a term in the one-loop correction to the Kähler potential. Instead, the leading term that breaks no-scale arises from the  $\alpha'$  correction in (6). Therefore, for a flux-induced superpotential of the form (2) the correction to the scalar potential is stronger suppressed at large volume than one could naively have expected from (6). It would be interesting to determine the effects of the Kähler potential (6) together with a non-perturbative superpotential with non-trivial  $\rho$ -dependence, along the lines of [8, 9].

With these observations the scalar potential that follows from setting  $D_S W = D_U W = 0$ , but  $D_{\rho} W \neq 0 \neq D_A W$  (so that supersymmetry is broken spontaneously), becomes

$$V = \frac{1}{(-i(\rho - \bar{\rho}))^3} \left[ \frac{c_1}{(-i(\rho - \bar{\rho}))^{3/2}} + \frac{c_2}{(-i(\rho - \bar{\rho}))^2} + \dots \right] |W|^2. \quad (8)$$

The prefactor comes from  $\exp(K)$ , and the coefficients  $c_1$  and  $c_2$  are functions of  $A$ , and depend on the values of the constants  $S^{(0)}$  and  $U^{(0)}$  that, in turn, depend on the flux

values. Note that to determine higher terms in the expansion in the inverse volume, one would also have to solve the relations  $D_S W = D_U W = 0$  for  $S$  and  $U$  to the next order in  $(\rho - \bar{\rho})^{-1}$ , and substitute into the scalar potential.

To find a minimum of the potential that stabilizes the volume, one should now minimize this potential with respect to  $A$  and  $\rho$ . As noted also in [12], in order to obtain a minimum at large values of the volume, one needs  $c_1 < 0$ ,  $c_2 > 0$  and

$$\left| \frac{c_2}{c_1} \right| \gg 1. \quad (9)$$

In a moment we will consider the concrete form of (8) for the  $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$  orientifold that we analyzed in [1].

### 3 The $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold

In this section we will review the known corrections to the Kähler potential as discussed in the previous section in a particular example, the  $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$  orientifold with D3- and D7-branes. We will see that the  $\alpha'$  correction in the scalar potential of the volume modulus (i.e. the term with coefficient  $c_1$  in the previous section) comes out with the wrong sign for volume stabilization in this model (more precisely for that version of this orientifold for which the perturbative corrections were computed in [1]). Nevertheless, we want to use it to give an impression of the qualitative features and the order of magnitude of the corrections coming from the one-loop corrections (i.e. of  $c_2$  of the last section), the reason being that the one-loop corrections are best understood in this particular case. We will come back to other models in the next section.

The supersymmetric Calabi-Yau-orbifold on  $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$  is defined through the elements in  $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{1, \Theta_1, \Theta_2, \Theta_3 = \Theta_1 \Theta_2\}$ , which all act on  $\mathbb{T}^6 = \mathbb{T}_1^2 \times \mathbb{T}_2^2 \times \mathbb{T}_3^2$  by reflection along four circles,  $\Theta_I$  leaving  $\mathbb{T}_I^2$  invariant and reflecting along the transverse  $\mathbb{T}_J^4$ ,  $I = 1, 2, 3$ . The world sheet parity projection that we consider is  $\Omega' = \Omega R_6 (-1)^{F_R}$ , with  $R_6$  the reflection of all six circles, and  $F_R$  the right-moving fermion number. The fixed loci of  $\Omega'$  and  $\Omega' \Theta_I$  define O3-planes and three sets of O7-planes. Accordingly, there are D3-branes and three sets of D7-branes, each of the latter wrapping one of the three  $\mathbb{T}_I^4$ .<sup>6</sup> Orientifold models based on  $\mathbb{Z}_2 \times \mathbb{Z}_2$  were the subject of much recent work on moduli stabilization [20, 21, 22, 23, 24].

---

<sup>6</sup>In [1] we employed the standard world sheet parity projection with  $\Omega$  instead of  $\Omega'$ , and thus worked with D9- and D5-branes. Here we will translate all results into the language of D3- and D7-branes by simple T-duality rules, as explained in [1].

The untwisted moduli of the model are three Kähler and complex structure moduli  $\{\rho^I, U^I\}$ , the axio-dilaton  $S$ , and the open string scalars. We will mostly restrict ourselves to a representative stack of D3-branes and denote its three complex position scalars as  $A^I$ , and we leave out the D7-brane scalars and twisted moduli.<sup>7</sup> We define the scalars as

$$\begin{aligned} S &= \frac{1}{\sqrt{8\pi^2}}(C + ie^{-\Phi}) , \quad U^I = \frac{1}{G_{44}^I}(G_{45}^I + i\sqrt{G^I}) , \quad A^I = a_4^I + U^I a_5^I , \\ \rho^I &= \frac{1}{\sqrt{8\pi^2}}(C_4^I|_{\mathbb{T}_I^4} + ie^{-\Phi}\mathcal{V}_{\mathbb{T}_I^4}) + \frac{1}{8\pi}A^I \frac{A^I - \bar{A}^I}{U^I - \bar{U}^I} . \end{aligned} \quad (10)$$

The classical (sphere plus disk level) Kähler potential of this model is

$$K^{(0)} = -\ln(S - \bar{S}) - \sum_{I=1}^3 \ln \left[ (\rho^I - \bar{\rho}^I)(U^I - \bar{U}^I) - \frac{1}{8\pi}(A^I - \bar{A}^I)^2 \right] . \quad (11)$$

Note that the moduli space does not factorize into Kähler moduli  $\rho^I$  and complex structure  $U^I$ , as long as  $\text{Im}(A^I) \neq 0$ .

The one-loop correction to the Kähler potential is given by (see equation (3.30) of [1])

$$K^{(1)} = \frac{1}{256\pi^6} \sum_{I=1}^3 \left[ \frac{\mathcal{E}_2^{\text{D3}}(A^I, U^I)}{(S - \bar{S})(\rho^I - \bar{\rho}^I)} + \frac{\mathcal{E}_2^{\text{D7}}(0, U^I)}{(\rho^J - \bar{\rho}^J)(\rho^K - \bar{\rho}^K)} \Big|_{K \neq I \neq J} \right] . \quad (12)$$

The superscripts “D3” and “D7” are used to indicate that the two terms require the presence of the respective type of Dp-brane in order to be non-vanishing, since they originate from open string diagrams with at least one boundary on these branes. Some comments are in order. In defining the function  $\mathcal{E}_2^{\text{D}p}(A, U)$ , we must ensure that a certain anomaly constraint on the position scalars of the D3-branes is satisfied. Until now, we have only kept a single D3-brane modulus  $A$  and ignored all other branes. However, the other scalars cannot be set to zero entirely, because the set of all scalars  $A_i$ ,  $i$  labelling the various stacks with gauge groups of rank  $N_i$  respectively, has to obey  $\sum_i N_i A_i = 0$  while  $\sum_i N_i = N_{\text{D3}}$ , where  $N_{\text{D3}}$  is the total D3-brane charge of the O3-planes. We write this as  $N_{\text{D3}} = 16 - N_{\text{flux}}$  in terms of the effective charge  $N_{\text{flux}}$  carried by background 3-form flux [2]<sup>8</sup>. The simplest solution is to consider three stacks with

<sup>7</sup>This effectively restricts us to a subset of fluxes — a generic flux destabilizes the orbifold limit such that the twisted moduli receive non-vanishing expectation values [25, 22, 26].

<sup>8</sup>This way of implementing the effect of 3-form flux on the 3-brane charge in the results of [1] is clearly slightly heuristic, since there was no flux considered in that calculation, and there may be additional corrections to the Kähler potential once fluxes are turned on, as we mentioned earlier.

$A_1 = -A_2 = A$ ,  $N_1 = N_2 = N$ , and  $A_3 = 0$ ,  $N_3 = N_{\text{D3}} - 2N$ . For the D7-branes we have set all scalars to zero from the beginning and there is no modification of the background 7-brane charge through fluxes. In this case one has (see equation (3.27) of [1])

$$\begin{aligned}\mathcal{E}_2^{\text{D3}}(A, U) &= 128N[E_2(A, U) + E_2(-A, U)] - 8N[E_2(2A, U) + E_2(-2A, U)] \\ &\quad + 120(N_{\text{D3}} - 2N)E_2(0, U) , \\ \mathcal{E}_2^{\text{D7}}(0, U) &= 1920 E_2(0, U) ,\end{aligned}\tag{13}$$

with

$$E_2(A, U) = \sum_{(n,m) \neq (0,0)} \frac{\text{Im}(U)^2}{|n + mU|^4} \exp \left[ 2\pi i \frac{A(n + m\bar{U}) - \bar{A}(n + mU)}{U - \bar{U}} \right] .\tag{14}$$

Note that  $E_2(A, U)$  is not holomorphic. Things get even simpler when setting  $2N = N_{\text{D3}}$ . In the special case when the D3-brane charge is completely cancelled by the 3-form flux, i.e.  $N_{\text{flux}} = 16$ , the first type of correction in (12) is absent. We show a plot of the function  $\mathcal{E}_2^{\text{D7}}(0, U)$  divided by  $256\pi^6$  in figures 1 and 2.

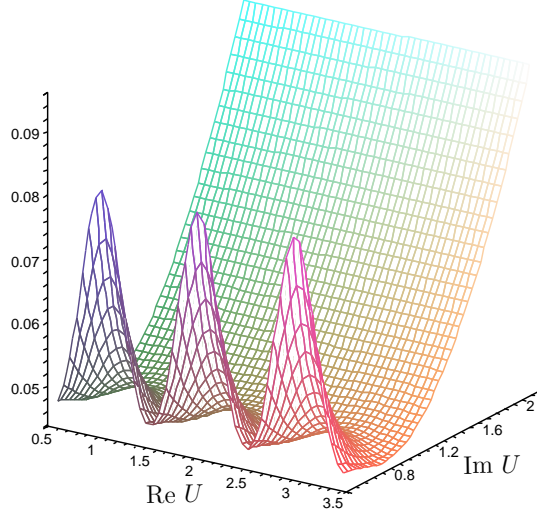


FIGURE 1: The expression  $\frac{1}{2}c\mathcal{E}_2^{\text{D7}}(0, U)$ , where  $c$  is given by  $(128\pi^6)^{-1}$  as in [1]. The oscillatory behavior for small imaginary part of  $U$  is shown in more detail in figure 2. For small real part and large imaginary part, the function behaves as  $\frac{1}{3}128\pi^4c \times \text{Im}(U)^2$  (see equation (B.3) in [1]).



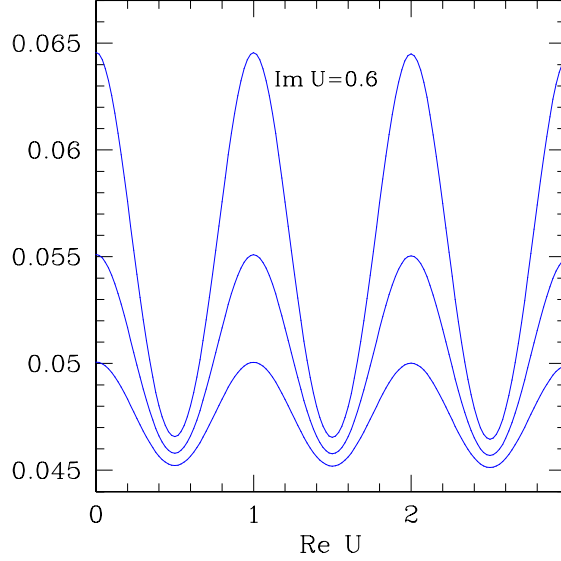


FIGURE 2: The expression  $\frac{1}{2}c\mathcal{E}_2^{D7}(0,U)$  along  $\text{Im}(U) \in \{0.6, 0.7, 0.8\}$  in figure 1.

In addition to the one-loop correction to the Kähler potential there is also the tree-level  $\alpha'$  correction of [11] here given by

$$K_{\alpha'}^{(0)} = \frac{\chi}{2}\zeta(3)\frac{(S-\bar{S})^{3/2}}{\sqrt{\prod_{I=1}^3(\rho^I - \bar{\rho}^I)}}. \quad (15)$$

This correction arises from sphere diagrams and thus does not depend on the open string moduli  $A^I$ . This completes the set of corrections to the Kähler potential known for this model, the full expression reading

$$K = K^{(0)} + K_{\alpha'}^{(0)} + K^{(1)}. \quad (16)$$

This presents a concrete realization of (6) with all coefficients determined. As mentioned above, this set of corrections may not be complete at the given order in the large volume expansion, due to the possibility of additional  $\alpha'$  corrections that could appear at tree-level onwards in the presence of fluxes.

The orientifold that we considered in [1] has gauge group  $Sp(8)^4$  and is T-dual to the model discussed in [27]. Thus, it has Hodge numbers  $(h^{(1,1)}, h^{(2,1)}) = (3, 51)$ , cf. [28, 29], and the Euler number is  $\chi = -96$ , the wrong sign to achieve volume stabilization along the lines outlined at the end of section 2. There do exist models with the right sign of the Euler number, such as  $\mathbb{T}^6/\mathbb{Z}'_6$  which we also considered in [1].

Here, the Kähler potential also exhibits one-loop correction terms that are suppressed by either  $(\rho - \bar{\rho})^{-1}$  or  $(\rho - \bar{\rho})^{-2}$ , such that the structure of that model is very similar to that of  $\mathbb{Z}_2 \times \mathbb{Z}_2$ .<sup>9</sup> Furthermore, we expect terms with the same volume dependence to appear also in other orientifold models, some of which would again have the right sign of the Euler number.<sup>10</sup>

Awaiting more detailed studies of corrections in other models, we continue by assuming we had indeed found a model with the right sign of the  $\alpha'$  correction and whose one-loop corrections are qualitatively captured by the formulas of the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  model discussed in this section, even though we do not have a concrete model at hand with the full set of corrections and all the coefficients determined. In this spirit, we will outline a situation where stabilization of the volume modulus is possible, but fine-tuning (of the complex structure modulus  $U$ ) is needed to obtain sufficiently large volume.

## 4 Perturbative volume stabilization

We now specialize to the case where all three tori are treated on equal footing, i.e. we set

$$\rho^1 = \rho^2 = \rho^3 \equiv \rho, \quad U^1 = U^2 = U^3 \equiv U, \quad A^1 = A^2 = A^3 \equiv A. \quad (17)$$

Then the Kähler potential becomes

$$\begin{aligned} K = & -\ln[S - \bar{S}] - 3\ln[(\rho - \bar{\rho})(U - \bar{U}) - f(A, \bar{A}, U, \bar{U})] \\ & + \alpha \frac{(S - \bar{S})^{3/2}}{(\rho - \bar{\rho})^{3/2}} + \frac{3c}{2} \frac{\mathcal{E}_2^{\text{D}3}(A, U)}{(S - \bar{S})(\rho - \bar{\rho})} + \frac{3c}{2} \frac{\mathcal{E}_2^{\text{D}7}(0, U)}{(\rho - \bar{\rho})^2} \end{aligned} \quad (18)$$

where

$$c = \frac{1}{128\pi^6}, \quad \alpha = \frac{\chi\zeta(3)}{2}. \quad (19)$$

We generalized the tree-level Kähler potential of  $\mathbb{Z}_2 \times \mathbb{Z}_2$  slightly by allowing for an arbitrary function  $f(A, \bar{A}, U, \bar{U})$  in the shift of the argument of the logarithm. For

---

<sup>9</sup>However, in that case we did not determine some of the exact numerical factors yet. Moreover, we cannot exclude that there are additional terms in the Kähler potential proportional to  $(\rho - \bar{\rho})^{-3/2}$  from additional one-loop contributions that are not present for  $\mathbb{Z}_2 \times \mathbb{Z}_2$ , but that we did not calculate in [1].

<sup>10</sup>An interesting candidate is the inequivalent version of the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orientifold with Hodge numbers  $(51, 3)$ . In the presence of world-volume gauge fluxes on the D-branes it allows supersymmetric solutions to the tadpole constraints (potentially with spontaneous supersymmetry breaking), see [20, 30].

$\mathbb{Z}_2 \times \mathbb{Z}_2$  it is  $f(A, \bar{A}, U, \bar{U}) = (8\pi)^{-1}(A - \bar{A})^2$ , and therefore independent of  $U$ . The resulting potential is

$$\begin{aligned} \frac{V}{e^K |W|^2} = & -\frac{3\alpha}{4} \frac{S_2^{3/2}}{\rho_2^{3/2}} + \left\{ \frac{3}{16} \frac{f^2}{U_2^2} - \frac{3i}{8} f(\partial_U - \partial_{\bar{U}})f + \frac{3c}{4} \mathcal{E}_2^{\text{D}7}(0, U) + \frac{3}{4} \partial_U f \partial_{\bar{U}} f \right. \\ & + \frac{3c}{8} \frac{1}{S_2} \left[ \frac{1}{2} \left( \frac{\mathcal{E}_2^{\text{D}3}(A, U)}{U_2} - i(\partial_U - \partial_{\bar{U}}) \mathcal{E}_2^{\text{D}3}(A, U) \right) f \right. \\ & \quad \left. \left. + U_2 \left( \partial_U \mathcal{E}_2^{\text{D}3}(A, U) \partial_{\bar{U}} f + \partial_{\bar{U}} \mathcal{E}_2^{\text{D}3}(A, U) \partial_U f \right) \right] \right. \\ & \left. + \frac{3c^2}{16} \frac{1}{S_2^2} \left[ U_2^2 \partial_U \mathcal{E}_2^{\text{D}3}(A, U) \partial_{\bar{U}} \mathcal{E}_2^{\text{D}3}(A, U) + \left( \mathcal{E}_2^{\text{D}3}(A, U) \right)^2 \right] \right\} \frac{1}{\rho_2^2} + \mathcal{O} \left( \frac{1}{\rho_2^{5/2}} \right), \end{aligned} \quad (20)$$

where  $\rho_2 = \text{Im}(\rho)$ , and so on. For the case that  $f(A, \bar{A}, U, \bar{U})$  is independent of  $U$ , and neglecting the terms proportional to  $(\text{Im}(S)\text{Im}(\rho)^2)^{-1}$  and  $(\text{Im}(S)\text{Im}(\rho))^{-2}$  that are suppressed for large values of  $\text{Im}(S)$ ,<sup>11</sup> this reduces to

$$\frac{V}{e^K |W|^2} = -\frac{3\alpha}{4} \frac{S_2^{3/2}}{\rho_2^{3/2}} + \frac{3}{4} \left( \frac{1}{4} \frac{f^2}{U_2^2} + c \mathcal{E}_2^{\text{D}7}(0, U) \right) \frac{1}{\rho_2^2} + \mathcal{O} \left( \frac{1}{\rho_2^{5/2}} \right). \quad (21)$$

Some comments are in order here. It is obvious that the no-scale structure is already broken by the tree-level part of the Kähler potential, once the argument of the logarithm is shifted by the function  $f(A, \bar{A}, U, \bar{U})$  and the factorization of the moduli space is lost. Furthermore, we see that we need a positive value for  $\alpha$  to obtain a negative sign for the  $\alpha'$  correction (i.e.  $c_1$  in formula (8)). According to (19) this translates into the requirement of a positive Euler number, which is unfortunately not met by the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orientifold in the version considered in [1].

As we already announced at the end of the last section, we continue heuristically, and ask what the prospects are for volume stabilization in a model with the right sign of the Euler number, under the assumption that its one-loop corrections are similar to those of the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orientifold. This is the case, for instance, for the  $\mathbb{Z}_6'$  orientifold. In this way we should be able to capture some of the qualitative features of volume stabilization by quantum corrections.<sup>12</sup>

Choosing  $\alpha = 50$  (corresponding to an Euler number of the order  $\chi \sim 111$ ) and assuming that  $f(A, \bar{A}, U, \bar{U}) = 0$  at the minimum (this is just for technical convenience and does not alter the result qualitatively), the potential does have a minimum and

<sup>11</sup>In any case, higher-genus loop corrections may contribute at those orders.

<sup>12</sup>It would also be interesting to better understand issues raised by [31] in this context.

the value of the volume at the minimum depends on the constants  $U^{(0)}$  and  $S^{(0)}$ , the leading terms of the large volume expansions of  $U$  and  $S$  at the minimum (we will drop the superscript (0) in the following) as

$$\text{Im}(\rho)|_{\min} \sim \frac{(\mathcal{E}_2^{\text{D7}}(0, U))^2}{(\text{Im}(S))^3} . \quad (22)$$

Using the form of  $E_2(A, U)$  (cf. (13) and (14)) given in (B.3) of [1], it follows that the volume (22) grows roughly as  $\text{Im}(U)^4$  for large  $\text{Im}(U)$  ( $\text{Im}(U) \gtrsim 5$ , say). Thus it is possible to obtain relatively large values for  $\text{Im}(\rho)$  at the minimum by tuning the values where  $U$  and  $S$  are stabilized. In practice it turns out that, to obtain a value of  $\text{Im}(\rho) = 100$  with  $\text{Im}(S) \sim 10$ , one has to resort to degenerate values for the complex structure of about  $\text{Im}(U) \sim 650$ , cf. figure 3. Since  $U$  is fixed through the flux superpotential to leading order, we would expect typical values of the order of 1 to 10, and thus consider  $\text{Im}(U) \sim 650$  a significant amount of fine-tuning. In addition, one would ultimately like to check whether higher-genus corrections might invalidate the genus expansion for such degenerate moduli values.

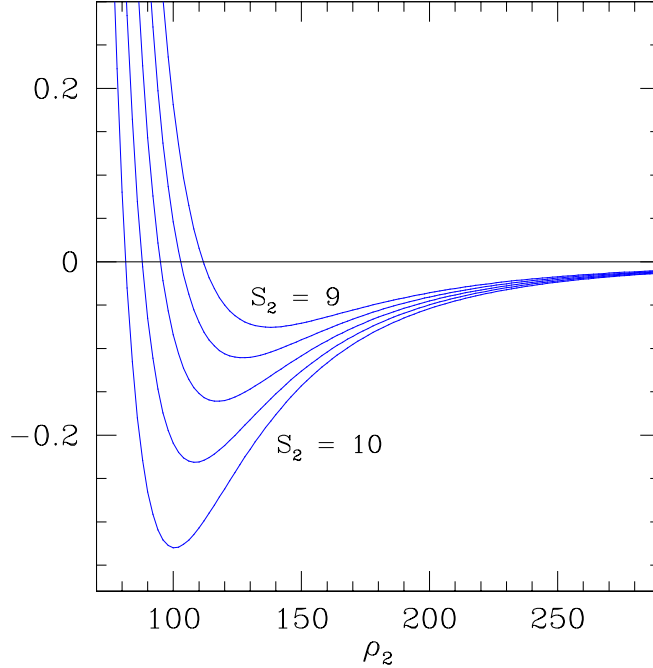


FIGURE 3: Volume stabilization with the potential (21), for a few values of  $\text{Im}(S)$ , and  $\text{Im}(U) = 650$ . The plot shows  $10^{18} \times V/|W|^2$ .

To summarize, the structure of the Kähler potential that arises in IIB orientifold compactifications including the known  $\alpha'$  and one loop corrections appears rich enough, in principle, to allow for purely perturbative stabilization of the volume modulus. The necessary properties for this to work, in the case we consider, are a positive Euler number and a tuning of the complex structure to large values. We have not been able to present a fully explicit model, but the  $\mathbb{Z}'_6$  orientifold comes close. Since it is not excluded that there exist additional corrections that cannot be neglected in the large volume and small coupling expansion, as was discussed above, care must be exercised in interpreting these results.

### Acknowledgements

We would like to thank Bogdan Florea, Arthur Hebecker, Jan Louis, Stephan Stieberger, and Tom Taylor for helpful advice and inspiring questions, during discussions and email correspondence. M.B. was supported by the Wenner-Gren Foundations, and M.H. by the German Science Foundation (DFG). Moreover, the research of M.B. and M.H. was supported in part by the National Science Foundation under Grant No. PHY99-07949. The work of B.K. was supported by the DFG, the DAAD, and the European RTN Program MRTN-CT-2004-503369, and in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818.

### References

- [1] M. Berg, M. Haack and B. Körs, *String Loop Corrections to Kähler Potentials in Orientifolds*, hep-th/0508043.
- [2] S. B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from Fluxes in String Compactifications*, Phys. Rev. D **66** (2002) 106006 [hep-th/0105097].
- [3] T. W. Grimm and J. Louis, *The Effective Action of  $N = 1$  Calabi-Yau Orientifolds*, Nucl. Phys. B **699**, 387 (2004) [hep-th/0403067].
- [4] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, *Naturally Vanishing Cosmological Constant in  $N=1$  Supergravity*, Phys. Lett. B **133** (1983) 61.
- [5] E. Witten, *New Issues in Manifolds of  $SU(3)$  Holonomy*, Nucl. Phys. B **268** (1986) 79.

- [6] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, *De Sitter Vacua in String Theory*, Phys. Rev. D **68** (2003) 046005 [hep-th/0301240].
- [7] V. Balasubramanian and P. Berglund, *Stringy Corrections to Kaehler Potentials, SUSY Breaking, and the Cosmological Constant Problem*, JHEP **0411** (2004) 085 [hep-th/0408054].
- [8] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, *Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications*, JHEP **0503**, 007 (2005) [hep-th/0502058].
- [9] J. P. Conlon, F. Quevedo and K. Suruliz, *Large-volume Flux Compactifications: Moduli Spectrum and D3/D7 Soft Supersymmetry Breaking*, hep-th/0505076.
- [10] I. Antoniadis, K. Benakli, A. Laugier and T. Maillard, *Brane to Bulk Supersymmetry Breaking and Radion Force at Micron Distances*, Nucl. Phys. B **662**, 40 (2003) [hep-ph/0211409].
- [11] K. Becker, M. Becker, M. Haack and J. Louis, *Supersymmetry Breaking and  $\alpha'$ -corrections to Flux Induced Potentials*, JHEP **0206**, 060 (2002) [hep-th/0204254].
- [12] G. von Gersdorff and A. Hebecker, *Kaehler Corrections for the Volume Modulus of Flux Compactifications*, hep-th/0507131.
- [13] K. Bobkov, *Volume Stabilization via  $\alpha'$  Corrections in Type IIB Theory with Fluxes*, JHEP **0505**, 010 (2005) [hep-th/0412239].
- [14] I. Antoniadis, C. Bachas, C. Fabre, H. Partouche and T. R. Taylor, *Aspects of Type I - Type II - Heterotic Triality in Four Dimensions*, Nucl. Phys. B **489** (1997) 160 [hep-th/9608012].
- [15] G. Aldazabal, A. Font, L. E. Ibanez and G. Violero,  *$D = 4$ ,  $N = 1$ , Type IIB Orientifolds*, Nucl. Phys. B **536** (1998) 29 [hep-th/9804026].
- [16] O. DeWolfe and S. B. Giddings, *Scales and Hierarchies in Warped Compactifications and Brane Worlds*, Phys. Rev. D **67** (2003) 066008 [hep-th/0208123].
- [17] H. Jockers and J. Louis, *The Effective Action of D7-branes in  $N = 1$  Calabi-Yau Orientifolds*, Nucl. Phys. B **705**, 167 (2005) [hep-th/0409098].
- [18] P. Berglund and P. Mayr, *Non-perturbative Superpotentials in F-theory and String Duality*, hep-th/0504058.

- [19] S. B. Giddings and A. Maharana, *Dynamics of Warped Compactifications and the Shape of the Warped Landscape*, hep-th/0507158.
- [20] R. Blumenhagen, D. Lüst and T. R. Taylor, *Moduli Stabilization in Chiral Type IIB Orientifold Models with Fluxes*, Nucl. Phys. B **663** (2003) 319 [hep-th/0303016].
- [21] J. P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, *Superpotentials in IIA Compactifications with General Fluxes*, Nucl. Phys. B **715** (2005) 211 [hep-th/0411276].
- [22] F. Denef, M. R. Douglas, B. Florea, A. Grassi and S. Kachru, *Fixing all Moduli in a Simple F-theory Compactification*, hep-th/0503124.
- [23] G. Villadoro and F. Zwirner,  *$N = 1$  Effective Potential from Dual Type-IIA D6/O6 Orientifolds with General Fluxes*, JHEP **0506** (2005) 047 [hep-th/0503169].
- [24] D. Lüst, S. Reffert, W. Schulgin and S. Stieberger, *Moduli Stabilization in Type IIB Orientifolds. I: Orbifold Limits*, hep-th/0506090.
- [25] D. Lüst, P. Mayr, S. Reffert and S. Stieberger, *F-theory Flux, Destabilization of Orientifolds and Soft Terms on D7-branes*, hep-th/0501139.
- [26] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA Moduli Stabilization*, JHEP **0507**, 066 (2005) [hep-th/0505160].
- [27] M. Berkooz and R. G. Leigh, *A  $D = 4$   $N = 1$  Orbifold of Type I Strings*, Nucl. Phys. B **483** (1997) 187 [hep-th/9605049].
- [28] I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti, *Open Descendants of  $Z(2) \times Z(2)$  Freely-acting Orbifolds*, Nucl. Phys. B **565**, 123 (2000) [hep-th/9907184].
- [29] M. Klein and R. Rabadan,  *$Z(N) \times Z(M)$  Orientifolds with and without Discrete Torsion*, JHEP **0010**, 049 (2000) [hep-th/0008173].
- [30] F. Marchesano and G. Shiu, *Building MSSM Flux Vacua*, JHEP **0411** (2004) 041 [hep-th/0409132].
- [31] E. Dudas, G. Pradisi, M. Nicolosi and A. Sagnotti, *On Tadpoles and Vacuum Redefinitions in String Theory*, Nucl. Phys. B **708** (2005) 3 [hep-th/0410101].